13-2 Probability with Permutations and Combinations

6. **CONCERTS** Nia and Chad are going to a concert with their high school's key club. If they choose a seat on the row below at random, what is the probability that Chad will be in seat C11 and Nia will be in C12?

SOLUTION:

Since choosing seats is a way of arranging the students, order in this situation is important. There are 12 seats. The number of possible outcomes in the sample space is the number of permutations of 12 seats taken 2 at a time, ${}_{12}P_2$.

$${}_{12}P_2 = \frac{12!}{(12-2)!} = \frac{12 \cdot 11 \cdot 10!}{10!} = 132$$

Among these, there is only one particular arrangement in which Chad will be in seat C11 and Nia will be in C12. Therefore, the probability is $\frac{1}{132}$.

ANSWER:

$\frac{1}{132}$

8. **CCSS MODELING** The table shows the finalists for a floor exercises competition. The order in which they will perform will be chosen randomly.

Floor Exercises Finalists
Eliza Hernandez
Kimi Kanazawa
Cecilia Long
Annie Montgomery
Shenice Malone
Caroline Smith
Jessica Watson

a. What is the probability that Cecilia, Annie, and Kimi are the first 3 gymnasts to perform, in any order?

b. What is the probability that Cecilia is first, Annie is second, and Kimi is third?

SOLUTION:

a. The number of possible outcomes is the number of arrangements of 7 performers taken 7 at a time. So, the number of possible outcomes is 7! = 5040.

The first 3 finalists can be arranged in 3! = 6 ways. The rest of the finalists can be arranged 4! ways.

Therefore, the number of favorable outcomes is(4!)(3!)= (24)(6) = 144.

The probability is $\frac{144}{5040} = \frac{1}{35}$.

b. The number of possible outcomes is 5040. The number of favorable outcomes with Cecilia in first place, Annie second, and Kimi in third place is 1. After the first three places are set, the rest of the finalists can be arranged 4! ways.

Therefore, the probability is $\frac{24}{5040} = \frac{1}{210}$.

ANSWER:

a. $\frac{1}{35}$

b. $\frac{1}{210}$

10. **GROUPS** Two people are chosen randomly from a group of ten. What is the probability that Jimmy was selected first and George second?

SOLUTION:

Since choosing people is a way of ranking the members, order in this situation is important. The number of possible outcomes in the sample space is the number of permutations of 10 people taken 2 at a time, ${}_{10}P_2$.

$${}_{18}P_3 = \frac{10!}{(10-2)!} = \frac{10 \cdot 9 \cdot 8!}{8!} = 90$$

Among these, there is only one particular arrangement in which Jimmy is selected first and George second. Therefore, the probability is $\frac{1}{90}$.

ANSWER:

$\frac{1}{90}$ or about 1%

12. **ZIP CODES** What is the probability that a zip code randomly generated from among the digits 3, 7, 3, 9, 5, 7, 2, and 3 is the number 39372?

SOLUTION:

There are 8 digits in which 3 appears three times and 7 appears twice.

The number of distinguishable permutations is $\frac{8!}{2! \cdot 3!} = 3360$.

The total number of possible outcomes is 3360 and there is only one favorable outcome which is 39372.

Therefore, the probability is $\frac{1}{3360}$.

ANSWER:

$\frac{1}{3360}$

14. **AMUSEMENT PARKS** Sylvie is at an amusement park with her friends. They go on a ride that has bucket seats in a circle. If there are 8 seats, what is the probability that Sylvie will be in the seat farthest from the entrance to the ride?

SOLUTION:

Since the people are seated with a fixed reference point, this is a linear permutation. So there are 8! or 40,320 ways in which the people can be seated. The number of favorable outcomes is the number of permutations of the other 7 people given that Sylvie will be in the seat farthest from the entrance to the ride, 7! or 5040.

Therefore, the probability is $\frac{5040}{40,320} = \frac{1}{8}$.

ANSWER:

$\frac{1}{8}$

16. **ROAD TRIPS** Rita is going on a road trip across the U.S. She needs to choose from 15 cities where she will stay for one night. If she randomly pulls 3 city brochures from a pile of 15, what is the probability that she chooses Austin, Cheyenne, and Savannah?

SOLUTION:

We are choosing 3 cities from a set of 15 cities and order is not important. So, the number of possible outcomes is ${}_{15}C_3$.

$${}_{15}C_3 = \frac{15!}{(15-3)!(3!)} = \frac{15 \cdot 14 \cdot 13 \cdot 12!}{12!(3 \cdot 2 \cdot 1)} = 455$$

The number of favorable outcomes is only one of choosing the cities Austin, Cheyenne, and Savannah.

Therefore, the probability is $\frac{1}{455}$.

ANSWER:

$\frac{1}{455}$

17. **CCSS SENSE-MAKING** Use the figure below. Assume that the balls are aligned at random.

a. What is the probability that in a row of 8 pool balls, the solid 2 and striped 11 would be first and second from the left?

b. What is the probability that if the 8 pool balls were mixed up at random, they would end up in the order shown?

c. What is the probability that in a row of seven balls, with three 8 balls, three 9 balls, and one 6 ball, the three 8 balls would be to the left of the 6 ball and the three 9 balls would be on the right?

d. If the balls were randomly rearranged and formed a circle, what is the probability that the 6 ball is next to the 7 ball?

SOLUTION:

a. The number of possible outcomes in the sample space is the number of permutations of 8 balls taken 8 at a time, 8! = 40,320.

The number of favorable outcomes is the number of permutations of the remaining 6 balls after fixing the solid 2 and striped 11 at the first and second from the left. 6! = 720

Therefore, the probability is $\frac{720}{40320} = \frac{1}{56}$.

b. The number of possible outcomes is 40,320 and the number of favorable outcome is the only one as shown. Therefore, the probability is $\frac{1}{40,320}$.

c. The required probability is to the probability of getting an arrangement of (8886999) from three 8 balls, three 9 balls and one 6 ball. So, the number of distinguishable permutations is $\frac{7!}{3!\cdot 3!} = 140$.

There is only one favorable outcome which is (8886999). Therefore, the probability is $\frac{1}{140}$.

d. Eight balls are arranged in the form of a circle with one fixed ball (7 ball). So, the circular permutation of arranging 8 balls with 1 fixed ball is 7!.

The 6 ball can be arranged either to the left of ball 7 or right of ball 7. So the circular permutation of arranging 8 balls with 2 fixed balls is $2 \cdot 6!$.

$$P(\text{6 ball is next to 7 ball}) = \frac{2 \cdot 6!}{7!} = \frac{2}{7}$$

ANSWER:

a. $\frac{1}{56}$

b. $\frac{1}{40,320}$

c. $\frac{1}{140}$

d. $\frac{2}{7}$

18. How many lines are determined by 10 randomly selected points, no 3 of which are collinear? Explain your calculation.

SOLUTION:

Two points determine a line. If no 3 points are collinear, then each line will consist of *exactly* 2 points. Since there are 10 total points, the number of lines is the combination of 10 points taken 2 at a time, which is $\frac{10!}{8!2!}$ or 45.

ANSWER:

45; Sample answer: The number of lines is the combination of 10 objects taken 2 at a time, which is $\frac{10!}{8!2!}$ or 45.

20. **RIDES** A carousel has 7 horses and one bench seat that will hold two people. One of the horses does not move up or down.

a. How many ways can the seats on the carousel be randomly filled by 9 people?

b. If the carousel is filled randomly, what is the probability that you and your friend will end up in the bench seat?

c. If 6 of the 9 people randomly filling the carousel are under the age of 8, what is the probability that a person under the age of 8 will end up on the one horse that does not move up or down?

SOLUTION:

a. The number of ways in which the 9 seats can be filled is the permutation of 9 people taken 9 at a time.

So, the number of ways is 9! = 362,880.

b. The total number of possible outcomes is 362,880. Since the carousel is filled with a fixed reference point, this is a linear permutation. So there are 7! or 5040 ways to arrange the remaining 7 people after you and your friend are placed in the bench seat. Also, for each arrangement there is an alternate arrangement by interchanging the position of you and your friend. Therefore, the probability is $\frac{2(5040)}{362{,}880} = \frac{1}{36}$.

c. The total number of arrangements of the 9 people is $\frac{9!}{6!}$ or 504. The number of favorable outcomes is the number of distinguishable permutations of the other eight places if a child under 8 is on the horse that does not move up or down: $\frac{8!}{5!}$ or 336. Calculate the probability.

$P(\text{child under 8 on broken horse}) = \frac{336}{504}$ or $\frac{2}{3}$

ANSWER:

a. 362,880

b. $\frac{1}{36}$

c. $\frac{2}{3}$

29. **PROBABILITY** Four members of the pep band, two girls and two boys, always stand in a row when they play. What is the probability that a girl will be at each end of the row if they line up in random order?

A. $\frac{1}{24}$

B. $\frac{1}{12}$

C. $\frac{1}{6}$

D. $\frac{1}{2}$

SOLUTION:

There are ${}_4P_4 = 24$ ways to arrange the band. Out of these, there are 2! or 2 ways to arrange the 2 boys in the middle of the two girls at the end and for each arrangement there is an alternate arrangement by interchanging the position of the two girls.

Therefore, the probability is $\frac{4}{24} = \frac{1}{6}$. The correct choice is C.

ANSWER:

C

30. **SHORT RESPONSE** If you randomly select a permutation of the letters shown below, what is the probability that they would spell GEOMETRY?

SOLUTION:

There are 8 letters in which only E appears twice and the others once each. So, the number of distinguishable permutations is $\frac{8!}{7(1!)\cdot 2!} = 20160$.

The total number of possible outcomes is 20160 and there is only one favorable outcome which is GEOMETRY. Therefore, the probability is $\frac{1}{20160}$.

ANSWER:

$\frac{1}{20160}$

31. **ALGEBRA** Student Council sells soft drinks at basketball games and makes $1.50 from each. If they pay $75 to rent the concession stand, how many soft drinks would they have to sell to make $250 profit?

F 116
G 117
H 167
J 217

SOLUTION:

Let x be the number of soft drinks that they have to sell to make a profit of $250.

$$1.50x = 250 + 75$$
$$1.50x = 325$$
$$x \approx 217$$

The correct choice is J.

ANSWER:

J

32. **SAT/ACT** The ratio of 12 : 9 is equal to the ratio of $\frac{1}{3}$ to __

A $\frac{1}{4}$
B 1
C $\frac{5}{4}$
D 2
E 4

SOLUTION:

Use the ratio to write and solve a proportion.

$$\frac{12}{9} = \frac{\frac{1}{3}}{x}$$
$$12x = 3$$
$$x = \frac{1}{4}$$

Therefore, the correct choice is A.

ANSWER:

A