CCSS REASONING Determine whether the events are *independent* or *dependent.* Then find the probability.

6. In a game, you roll an even number on a die and then spin a spinner numbered 1 through 5 and get an odd number.

SOLUTION:

Since the probability of the fist event does not affect the probability of the second event, these are independent events.

$P(\text{even number on die}) = \frac{3}{6}$

$P(\text{odd number on spinner}) = \frac{3}{5}$

If two events *A* and *B* are independent, then *P*(*A* and *B*) = *P*(*A*) · *P*(*B*).

$$\begin{aligned} P(\text{even/odd}) &= P(\text{even-die}) \cdot P(\text{odd-spinner}) \\ &= \frac{3}{6} \cdot \frac{3}{5} \\ &= \frac{9}{30} \\ &= \frac{3}{10} \\ &= 30\% \end{aligned}$$

ANSWER:

independent; $\frac{3}{10}$ or 30%

8. In a bag of 3 green and 4 blue marbles, a blue marble is drawn and not replaced. Then, a second blue marble is drawn.

SOLUTION:

These events are dependent since a blue marble is not replaced before the second draw.

If two events *A* and *B* are dependent, then *P*(*A* and *B*) = *P*(*A*) · *P*(*B*|*A*).

$$\begin{aligned} P(\text{blue and blue}) &= P(\text{blue}) \cdot P(\text{blue}|\text{blue}) \\ &= \frac{4}{7} \cdot \frac{3}{6} \\ &= \frac{2}{7} \\ &\approx 29\% \end{aligned}$$

ANSWER:

dependent; $\frac{2}{7}$ or about 29%

10. **GAMES** In a game, the spinner is spun and a coin is tossed. What is the probability of getting an even number on the spinner and the coin landing on tails?

SOLUTION:

Since the probability of the first event does not affect the probability of the second event, these are independent events.

$P(\text{even}) = \frac{3}{6}$

$P(\text{tails}) = \frac{1}{2}$

If two events *A* and *B* are independent, then *P*(*A* and *B*) = *P*(*A*)*P*(*B*).

$$P(\text{even and tails}) = P(\text{even}) \cdot P(\text{tails})$$
$$= \frac{3}{6} \cdot \frac{1}{2}$$
$$= \frac{1}{4}$$
$$= 25\%$$

ANSWER:

$\frac{1}{4}$ or 25%

12. **VACATION** A work survey found that 8 out of every 10 employees went on vacation last summer. If 3 employees' names are randomly chosen, with replacement, what is the probability that all 3 employees went on vacation last summer?

SOLUTION:

Since the probability of the fist event does not affect the probability of the second event, these are independent events. Let *V* represent selecting people who went on vacation last summer.

$$P(V) = \frac{8}{10} \cdot \frac{8}{10} \cdot \frac{8}{10}$$
$$= \frac{512}{1000}$$
$$= 51.2\%$$
$$\approx 51\%$$

ANSWER:

$\frac{512}{1000}$ or about 51%

14. A red marble is selected at random from a bag of 2 blue and 9 red marbles and not replaced. What is the probability that a second marble selected will be red?

SOLUTION:

A red marble is selected at random. The chosen marble is not replaced before the second draw. Therefore, the number of red marbles becomes 8 and the total number of marbles in the bag is 10.
The probability of drawing second red marble of 8 red marbles out of 10 marbles is $\frac{8}{10}$.

$$P(\text{another red}) = \frac{8}{10}$$
$$= \frac{4}{5}$$
$$= 80\%$$

ANSWER:

$\frac{4}{5}$ or 80%

16. A quadrilateral has a perimeter of 12 and all of the side lengths are odd integers. What is the probability that the quadrilateral is a rhombus?

SOLUTION:

Possible outcomes { (1, 3, 5, 3), (1, 1, 5, 5), (3, 3, 3, 3), (1, 1, 1, 9), (7, 1, 1, 3)}
Number of possible outcomes: 5
Favorable outcomes: {(3, 3, 3, 3)}
Number of favorable outcomes: 1
The probability of getting the quadrilateral is a rhombus is $\frac{1}{5}$.

$$P(\text{rhombus}) = \frac{1}{5} = 20\%.$$

ANSWER:

$\frac{1}{5}$ or 20%

18. **CLASSES** The probability that a student takes geometry and French at Satomi's school is 0.064. The probability that a student takes French is 0.45. What is the probability that a student takes geometry if the student takes French?

SOLUTION:

$$\begin{aligned} P(\text{g}|\text{f}) &= \frac{P(\text{g and f})}{P(\text{f})} \\ &= \frac{0.064}{0.45} \\ &\approx 0.14 \end{aligned}$$

ANSWER:

0.14

20. **PROOF** Use the formula for the probability of two dependent events $P(A \text{ and } B)$ to derive the conditional probability formula for $P(B|A)$.

SOLUTION:

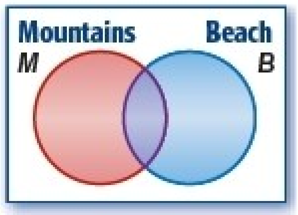
Use the formula for the probability of two dependent events to derive the formula for finding the conditional probability that an event occurs given that another event as already occurred.

$$\begin{aligned} P(A \text{ and } B) &= P(A) \cdot P(B|A) && \text{Formula for } P(A \text{ and } B) \\ \frac{P(A \text{ and } B)}{P(A)} &= P(B|A) && \text{Divide each side by } P(A). \end{aligned}$$

ANSWER:

$$\begin{aligned} P(A \text{ and } B) &= P(A) \cdot P(B|A) && \text{Formula for } P(A \text{ and } B) \\ \frac{P(A \text{ and } B)}{P(A)} &= P(B|A) && \text{Divide each side by } P(A). \end{aligned}$$

22. **VACATION** A random survey was conducted to determine where families vacationed. The results indicated that $P(B) = 0.6$, $P(B \cap M) = 0.2$, and the probability that a family did not vacation at either destination is 0.1.

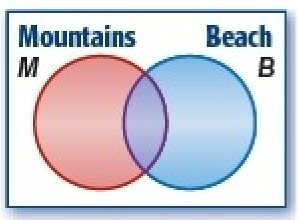


a. What is the probability that a family vacations in the mountains?
b. What is the probability that a family that has already visited the beach will also visit the mountains?

SOLUTION:

a. We are given $P(B)$, $P(B \cap M) = 0.2$, and P (neither).

Refer to the diagram.



$P(B)$ [the entire blue circle] = 0.6

$P(B \cap M)$ [the diamond-shaped intersection of the two circles] is the probability that they vacationed at both places. $P(B \cap M) = 0.2$.

P(neither) [the white background] = 0.1

If P(neither) = 0.1 and $P(B) = 0.6$, then $P(M) = 1 - (0.1 + 0.6) = 0.3$. Remember that the sum of the probabilities is 1.

b. We are looking for $P(B|M)$.

$$P(B|M) = \frac{P(B \text{ and } M)}{P(M)} = \frac{0.2}{0.6} \approx 0.33$$

ANSWER:

a. 0.3
b. 0.33

24. **CCSS ARGUMENTS** There are n different objects in a bag. The probability of drawing object A and then object B without replacement is about 2.4%. What is the value of n? Explain.

SOLUTION:

When there are n objects, the probability of randomly selecting one of the objects, object A, is $\frac{1}{n}$.

Since A was not replaced, there are now $n - 1$ objects left and the probability of randomly selecting object B is $\frac{1}{n-1}$.

$P(A \text{ and } B) = P(A) \cdot P(B \mid A)$	Formula for probability of dependent events
$0.024 = \frac{1}{n} \cdot \frac{1}{n-1}$	Substitution
$0.024(n)(n-1) = (n)(n-1)\left(\frac{1}{n} \cdot \frac{1}{n-1}\right)$	Multiply each side by $n(n-1)$.
$0.024n^2 - 0.024n = 1$	Simplify.
$0.024n^2 - 0.024n - 1 = 0$	Subtract 1 from each side.
$24n^2 - 24n - 1000 = 0$	Multiply each side by 1000.
$3n^2 - 3n - 125 = 0$	Divide each side by 8.

Use the quadratic formula to find the value of n.

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	Quadratic Formula
$n = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(3)(-125)}}{2(3)}$	$a = 3, b = -3, c = -125$
$n = \frac{3 \pm \sqrt{1509}}{6}$	Simplify.
$n = \frac{3 + \sqrt{1509}}{6}$	The number of objects must be positive.
$n \approx 6.974$	Use a calculator.

The number of objects is an integer value, so the value of n must equal 7.

ANSWER:

7; Sample answer: The probability of drawing object A is $\frac{1}{n}$, and the probability of drawing object B when object A is not replaced is $\frac{1}{n-1}$. Since we know that the probability is 2.4%, $\frac{1}{n} \cdot \frac{1}{n-1} = \frac{2.4}{100}$ or 0.024. Solve this equation to determine that n is 7.

26. **OPEN ENDED** Describe a pair of independent events and a pair of dependent events. Explain your reasoning.

SOLUTION:

Think about the difference between dependent and independent events. What sets them apart?

Sample answer: Flipping a coin two times represents an independent event. Regardless of the outcome of the first flip, the probability of getting heads and tails on the second flip does not change. Drawing two colored marbles out of a bag without replacing the first marble represents a dependent event. Based on the color of the first marble, the probability that the second marble will be a specific color will change.

ANSWER:

Sample answer: Flipping a coin two times represents an independent event. Regardless of the outcome of the first flip, the probability of getting heads and tails on the second flip does not change. Drawing two colored marbles out of a bag without replacing the first marble represents a dependent event. Based on the color of the first marble, the probability that the second marble will be a specific color will change.

28. **PROBABILITY** Shannon will be assigned at random to 1 of 6 P.E. classes throughout the day and 1 of 3 lunch times. What is the probability that she will be in the second P.E. class and the first lunch?

A $\frac{1}{18}$

B $\frac{1}{9}$

C $\frac{1}{6}$

D $\frac{1}{2}$

SOLUTION:

Since the probability of the fist event does not affect the probability of the second event, these are independent events.

$P(\text{second P.E. class}) = \frac{1}{6}$

$P(\text{first lunch}) = \frac{1}{3}$

If two events *A* and *B* are independent, then $P(A \text{ and } B) = P(A)P(B)$.

$$P(\text{PE and lunch}) = P(\text{PE}) \cdot P(\text{lunch})$$

$$= \frac{1}{6} \cdot \frac{1}{3}$$

$$= \frac{1}{18}$$

So, the correct choice is A.

ANSWER:

A

30. **GRIDDED RESPONSE** A bag contains 7 red jelly beans, 11 yellow jelly beans, and 13 green jelly beans. Victoro picks two jelly beans from the bag without looking. What is the probability that Victoro picks a green one and then a red one? Write the probability as a percent rounded to the nearest tenth.

SOLUTION:

The probability of drawing first green jelly bean of 13 green jelly beans out of 31 jelly beans is

$P(\text{green}) = \frac{13}{31}$.

The chosen jelly bean is not replaced before the second draw. Therefore, the total number of jelly beans in the bag is 30.
The probability of drawing second red jelly bean of 7 red jelly beans out of 30 jelly beans is $P(\text{red}) = \frac{7}{30}$.

$$\begin{aligned} P(\text{green, red}) &= P(\text{green}) \times P(\text{red}) \\ &= \frac{13}{31} \times \frac{7}{30} \\ &\approx 0.0978 \\ &\approx 9.8\% \end{aligned}$$

ANSWER:
9.8