Chapter 13

1. **FITNESS** Laura wants to go to a fitness class tomorrow. She can choose a 5:00 or a 7:30 class and spin or water aerobics. Represent the sample space for the situation by making an organized list, a table, and a tree diagram.

SOLUTION:

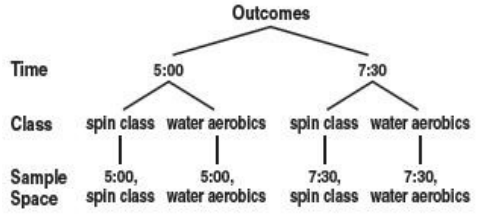
There are 2 times and 2 types of classes. List every combination.

5:00, spin class
5:00, water aerobics
7:30, spin class
7:30, water aerobics

List the times in the rows and the types in the columns.

Outcomes	spin class	water aerobics
5:00	5:00, spin class	5:00, water aerobics
7:30	7:30, spin class	7:30, water aerobics

List the times first. For each time, connect to every possible type.



ANSWER:

5:00, spin class
5:00, water aerobics
7:30, spin class
7:30, water aerobics

Outcomes	spin class	water aerobics
5:00	5:00, spin class	5:00, water aerobics
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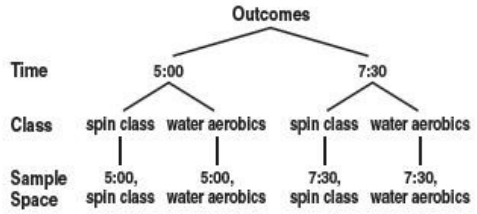
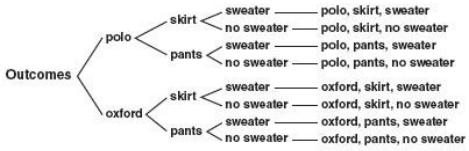


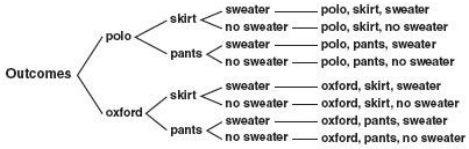
2. **SCHOOL UNIFORMS** Susan has a school uniform that consists of a polo shirt, an oxford shirt, a skirt, and a pair of pants. She also has a sweater than she can wear if she chooses. Draw a tree diagram to represent the sample space for Susan's uniform.

SOLUTION:

List the type of shirt first. For each type, connect to skirt and pants. For each of these connections, connect to *sweater* and *no sweater*.



ANSWER:



3. **CONSTRUCTION** Bert's family is building a house in a new neighborhood, and they must choose one option listed below for each feature. What is the number of possible outcomes for the situation?

Feature	Options
floor plan	Elevation 1, Elevation 2
counters	formica, granite
cabinets	French antique glazed, oak, cherry
basement	unfinished, partially finished, finished
garage	none, one car, two car

SOLUTION:

There are 2 floor plans. There are 2 counter types. There are 3 cabinets. There are 3 basements. There are 3 types of garages. Multiply each of these values together.

$2 \times 2 \times 3 \times 3 \times 3 = 108$

ANSWER:

108

4. **DIVING** At a swim meet, the order of the divers is randomly selected. If there are 12 divers, what is the probability that Danielle, Nora, and Li will dive first, second, and third, respectively?

SOLUTION:

$$\frac{1}{12} \cdot \frac{1}{11} \cdot \frac{1}{10} = \frac{1}{1320}$$

ANSWER:

$\frac{1}{1320}$

5. **NUMBERS** Charlie's phone number is 555–3703. If he places each of the digits in a bowl and randomly selects one number at a time without replacement, what is the probability that he will choose his phone number?

SOLUTION:

There are 7 digits, with 3 5s and 2 3s. Find the individual probabilities then multiply.

Do the 1st digit, then the 2nd digit, then so on.

$$\frac{3}{7} \cdot \frac{2}{6} \cdot \frac{1}{5} \cdot \frac{2}{4} \cdot \frac{1}{3} \cdot \frac{1}{2} = \frac{12}{5040} = \frac{1}{420}$$

ANSWER:

$\frac{1}{420}$

6. **RAFFLES** Participants in a raffle received tickets 1101 through 1125. If four winners are chosen, what is the probability that the winning tickets are 1103, 1111, 1118, and 1122?

SOLUTION:

There are 25 tickets and 4 are drawn. The order in which they are drawn does not matter, so this is a combination.

$${}_{25}C_4 = 12,650$$

The probability is $\frac{1}{12,650}$.

ANSWER:

$\frac{1}{12,650}$

7. Point X is chosen at random on $\overline{AE}$. Find the probability that X is on $\overline{CE}$.

SOLUTION:

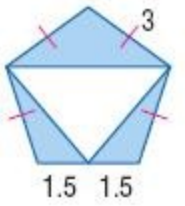
$$P = \frac{CE}{AE} = \frac{5+8}{4+7+5+8} = \frac{13}{24} \approx 54\%$$

ANSWER:

about 0.54 or 54%

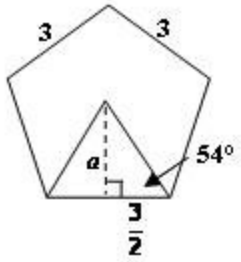
Find the probability that a point chosen at random lies in the shaded region.

8.



SOLUTION:

Use a trigonometric ratio to find the apothem of the regular pentagon. Remember that the measure of each interior angle of the regular pentagon is 108 degrees and half of that is 54 degrees.



$$\tan 54 = \frac{a}{\frac{3}{2}}$$

$$\frac{3}{2}\tan 54 = a$$

Next, find the area of the regular pentagon.

$$A = \frac{1}{2}aP$$

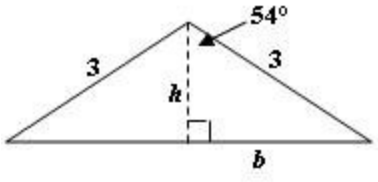
$$A = \frac{1}{2}\left(\frac{3}{2}\tan 54\right)(5 \times 3)$$

$$A \approx \frac{1}{2}(2.06)(15)$$

$$A \approx 15.48$$

Now find the area of the shaded regions.

We can use trigonometry to find the base and the height of the top shaded triangle since we know the measure of the interior angle is 108.



$$\cos 54 = \frac{h}{3}$$
$$3\cos 54 = h$$

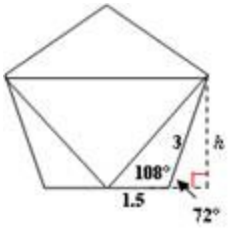
$$\sin 54 = \frac{b}{3}$$
$$3\sin 54 = b$$

The base of the full triangle is 2*b* or 6 sin 54.

Find the area of the top triangle.

$$\begin{aligned} A &= \frac{1}{2}bh \\ &= \frac{1}{2}(6\sin 54)(3\cos 54) \\ &= 9\sin 54\cos 54 \\ &\approx 4.28 \end{aligned}$$

Next, use a trigonometric ratio to find the height for one of the two congruent shaded triangles on the bottom.



$$\sin 72 = \frac{h}{3}$$
$$3\sin 72 = h$$

Now find the area of each shaded triangle at the bottom.

$$\begin{aligned} A &= \frac{1}{2}bh \\ &= \frac{1}{2}(1.5)(3\sin 72) \\ &\approx 2.14 \end{aligned}$$

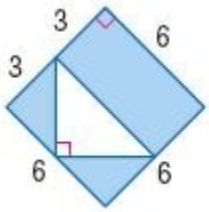
Thus, the area of the shaded regions is 2.14 + 2.14 + 4.28 or 8.56.

$$\begin{aligned} P(\text{shaded}) &= \frac{Area(shaded)}{Area(pentagon)} \\ &\approx \frac{8.56}{15.48} \\ &\approx 0.553 \text{ or } 55.3\% \end{aligned}$$

So, the probability that a point chosen at random lies in the shaded region is about 0.553 or 55.3%.

ANSWER:
≈ 0.553 *or* 55.3%

9.



SOLUTION:
The base and height of the white triangle are each the hypotenuse of a 45-45-90 triangle with legs of 3. The length of the height of the white triangle is $3\sqrt{2}$ and the length of the base is $3\sqrt{2}$.

The area of the triangle is $0.5\left(3\sqrt{2}\right)\left(3\sqrt{2}\right) = 9$.

The area of the square is 6 × 6 = 36.

The area of the shaded region is the area of the square minus the area of the white triangle or 36 – 9 = 27.

The probability that a point chosen at random lies in the shaded region is $\frac{27}{36} = 0.75$ or 75%.

ANSWER:
0.75 or 75%

10.

SOLUTION:

The unmarked shaded region is 360 – 331 = 29.

The probability is $\frac{29+48+73}{360} = \frac{150}{360} = \frac{5}{12} \approx 41.7\%$.

ANSWER:

$\frac{5}{12}$ *or about* 41.7%

11. **FOOTBALL** Wes made 92% of his point after touchdown attempts last season. Design and conduct a simulation using a random number generator that can be used to estimate the probability that he will make his next point after touchdown attempt.

SOLUTION:

92% is not a good percentage for drawing cards, spinning spinners, or rolling dice. A random number generator is the most effective.

Sample answer: Use a random number generator to generate integers 1 through 100, where 1 – 92 represent a successful attempt, and 93 – 100 represent an unsuccessful attempt. Do 50 trials and record the results in a frequency table.

Outcome	Frequency
successful	46
unsuccessful	4

The probability that Wes' next point after touchdown attempt is successful is 0.92 or 92%, which is the same as the theoretical probability.

ANSWER:

Sample answer: Use a random number generator to generate integers 1 through 100, where 1 – 92 represent a successful attempt, and 93 – 100 represent an unsuccessful attempt. Do 50 trials and record the results in a frequency table.

Outcome	Frequency
successful	46
unsuccessful	4

The probability that Wes' next point after touchdown attempt is successful is 0.92 or 92%, which is the same as the theoretical probability.

BASKETBALL For each field goal attempt in basketball, a player can earn 0, 2, or 3 points. The probability that a certain player will score 0 points on an attempt is 45%, 2 points is 40%, and 3 points is 15%.

12. Calculate the expected value for one attempt.

SOLUTION:

$0(0.45) + 2(0.40) + 3(0.15) = 0 + 0.80 + 0.45 = 1.25$

ANSWER:

1.25

13. Design a simulation using a geometric probability model and estimate the player's average value per field goal attempt.

SOLUTION:

While a spinner is a possible form of simulation, a number generator would be much more effective and easier to use. For the generator, generate numbers from 1-100, with 1-45 representing 0 points, 46-85 representing 2 points, and 86-100 representing 3 points.

Sample answer: Use a spinner divided into sectors with angle measures of 162° to represent 0 points, 144° to represent 2 points, and 54° to represent 3 points. Do 20 trials and record the results in a frequency table.

Outcome	Frequency
0 points	8
2 points	11
3 points	1

The average value is 1.15 points per attempt.

ANSWER:

Sample answer: Use a spinner divided into sectors with angle measures of 162° to represent 0 points, 144° to represent 2 points, and 54° to represent 3 points. Do 20 trials and record the results in a frequency table.

Outcome	Frequency
0 points	8
2 points	11
3 points	1

The average value is 1.15 points per attempt.

14. Compare the values for Exercises 12 and 13.

SOLUTION:

1.15 is relatively close to 1.25.

ANSWER:

Sample answer: The values are about the same.

15. A die is rolled twice. What is the probability that the first number rolled is a 3 and the second number rolled is a 5?

SOLUTION:

$$\frac{\text{rolliunga 3}}{\text{6options}} \cdot \frac{\text{rollinga 5}}{\text{6options}} = \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36}$$

ANSWER:

$\frac{1}{36}$

16. Three cards are randomly chosen from a deck of 52 cards without replacement. What is the probability that they will all be red?

SOLUTION:

$$\frac{26red}{52total} \cdot \frac{25red}{51total} \cdot \frac{24red}{50total} = \frac{15{,}600}{132{,}600} \approx 11.8\%$$

ANSWER:

about 0.118 or 11.8%

17. A spinner numbered 1 through 6 is spun. Find the probability that the number spun is a 3 given that it was less than 4.

SOLUTION:

$$\frac{\text{spinninga 3}}{1or\ 2or\ 3} = \frac{1}{3}$$

ANSWER:

$\frac{1}{3}$

BOOKS The table shows the number and type of books that Sarah owns. Find each probability.

Medium	Classic	Mystery	Biography
print	29	8	32
audio	3	6	10
electronic	8	3	43

18. A randomly chosen title is a print or audio book.

SOLUTION:

Find the sum of the print row and the audio row. Divide the sum of these two values by the total number of books.

$$\frac{29+8+32+3+6+10}{29+8+32+3+6+10+8+3+43} = \frac{88}{142} \approx 62\%$$

ANSWER:

about 0.62 or 62%

19. A randomly chosen title is not a biography.

SOLUTION:

Find the sum of the Classic and Mystery columns. Divide the sum of these two values by the total number of books.

$$\frac{29+3+8+8+6+3}{29+8+32+3+6+10+8+3+43} = \frac{57}{142} \approx 40.1\%$$

ANSWER:

about 0.401 or 40.1%

20. **DOGS** The table shows the ages and genders of the dogs at an animal shelter. What is the probability that a randomly chosen dog is a female or over 5 years old?

Age	Male	Female
under 1 year	6	5
1–5 years	8	7
6–10 years	4	6
over 10 years	3	5

SOLUTION:

There are 23 females. There are 18 over 5 years old. There are 11 that are female and over 5 years old.

So, there are 23 + 18 – 11 = 30 that are female or over 5 years old.

There are 44 total dogs.

Therefore, there are 30 ÷ 44 = 68% that are female or over 5 years old.

ANSWER:

about 0.68 or 68%